

An enumeration of certain projective ternary two-weight codes and their relationship to the cubic Segre variety

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We have dedicated this work to the memory of Axel Kohnert (1962 – 2013).

ABSTRACT. We detail the enumeration of all two-intersection sets of the five-dimensional projective space over the field of order 3 that are invariant under an element of order 7, which include the examples of Hill (1973) and Gulliver (1996). Up to projective equivalence, there are 6635 such two-intersection sets.

1. Introduction

In the early 1970's, Philippe Delsarte showed that there is a remarkable connection between three objects in mathematics:

- (i) strongly regular graphs;
- (ii) linear codes with two weights;
- (iii) sets of points in a projective space with two intersection sizes with respect to hyperplanes.

To illustrate these connections, consider the five-dimensional projective space over the field of order 3. There are some captivating examples of two-intersection sets in this space, one of which was discovered by Ray Hill [8] in 1973 and it is known as *Hill's 56-cap* since it consists of 56 points (of $\text{PG}(5, 3)$), no three lying on a common line. This cap has the property that any hyperplane of $\text{PG}(5, 3)$ has only two possible ways of intersecting the cap: in 11 or 20 points. Equivalently, we can associate the points of this cap with the columns of a generator matrix for a ternary linear code with parameters $[56, 6]$ and weights 36 and 45. Using a beautiful geometric construction known as *linear representation*, Hill's 56-cap gives rise to a geometry known as a *partial quadrangle*, and it therefore gives rise to a strongly regular graph on 729 vertices. Today, we also know that Hill's 56-cap is intimately linked to Segre's hemisystem of the Hermitian variety $\text{H}(3, 3^2)$ and to an interesting imprimitive cometric Q -antipodal 4-class association scheme [14]. There are other examples in this space found by Gulliver [7], and these examples have something in common; they each admit a common symmetry of order 7, which happens to be $q^2 - q + 1$ when $q = 3$. In general, when $q \not\equiv 2 \pmod{3}$, we can *factor out* this symmetry and identify these two-intersection sets with subsets of the classical algebraic variety known as the *cubic Segre variety* (see Section 3).

This paper details the search for, and enumeration of, a family of strongly regular graphs arising from two-intersection sets of $\text{PG}(5, 3)$. Of the three mathematical objects above, two-intersection sets are the most readily computable, and hence effort was focused on finding two-intersection sets using linear programming techniques. In particular, the projective space $\text{PG}(5, 3)$ was examined as a successor to $\text{PG}(5, 2)$, since the binary projective codes of dimension at most 6 have been classified by Bouyukliev [1]. Over 6000 new strongly regular graphs were discovered as a result

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of enumerating two-weight ternary codes of dimension 6, many of which overlap the list of codes on Kohnert's webpage¹. Up to projective equivalence, there are 6635 such two-intersection sets of $\text{PG}(5, 3)$ invariant under a collineation of order 7.

2. Correspondences between two-weight linear codes, two-intersection sets and strongly regular graphs

We begin with the elementary definitions of all three objects, and the relationships between them. An $[n, k]$ *linear code* is a k -dimensional subspace of \mathbb{F}_q^n , and its vectors are known as *codewords*. The weight of a codeword is the number of non-zero values in its coordinate vector. Every $[n, k]$ linear code is the row space of some $k \times n$ generator matrix. A linear code C is *projective* if no two columns of its generator matrix are linearly dependent. Equivalently, C is projective if the minimum weight of the dual code C^\perp of C is at least 3. A two-weight linear code is simply a linear code in which all non-zero codewords have weight one of two fixed values w_1 or w_2 . The family of two-weight linear codes is an important one; a 2-error-correcting linear code is perfect if and only if its dual is a two-weight code, and a 1-error-correcting linear code is uniformly packed if and only if its dual is a two-weight code [2, Theorem 4.2, Theorem 4.3].

A two-intersection set \mathcal{K} with parameters (n, k, h_1, h_2) is a set of points in $\text{PG}(k-1, q)$ such that every hyperplane in $\text{PG}(k-1, q)$ is incident with either h_1 or h_2 points in \mathcal{K} . For example, the points lying on a line form a simple two-intersection set with parameters $(q+1, k, 1, q+1)$: every hyperplane either lies completely on the line (and hence intersects it at $q+1$ points), or meets it at one point. The complement of a two-intersection set with parameters (n, k, h_1, h_2) is again a two-intersection set (but with different parameters).

Every projective two-weight linear code arises from a two-intersection set and gives rise to a strongly regular graph. A strongly regular graph with parameters (n, k, λ, μ) is a graph containing n vertices in which every vertex has degree k , every two adjacent vertices share λ common neighbours, and every two non-adjacent vertices share μ common neighbours. The following results were originally proved by Delsarte [3], and provide the necessary details for converting a two-intersection set into a two-weight linear code and for converting a two-weight linear code into a two-intersection set or a strongly regular graph.

THEOREM 2.1 (c.f., Calderbank and Kantor [2, Theorem 3.2]). *If $\mathcal{K} = \{p_1, p_2, \dots, p_n\}$ is a two-intersection set with parameters (n, k, h_1, h_2) that spans $\text{PG}(k-1, q)$, then*

$$\mathcal{G} = [p_1^\perp \quad p_2^\perp \quad \cdots \quad p_n^\perp]$$

is the generator matrix of a projective two-weight $[n, k]$ linear code with weights $n - h_1$ and $n - h_2$, where it is understood that the p_i are vectors representing the homogeneous coordinates of points of $\text{PG}(k-1, q)$. Conversely, if $\mathcal{G} = [c_1 \ c_2 \ \cdots \ c_n]$ is the generator matrix of a projective two-weight $[n, k]$ linear code with weights w_1 and w_2 , then $\mathcal{K} = \{c_1^\perp, c_2^\perp, \dots, c_n^\perp\}$ are the homogeneous coordinate vectors for a two-intersection set with parameters $(n, k, n - w_1, n - w_2)$ that spans $\text{PG}(k-1, q)$.

Delsarte defined the associated graph $\Gamma(\mathcal{C})$ of a projective two-weight linear code \mathcal{C} with weights w_1 and w_2 as follows: let the vertices of $\Gamma(\mathcal{C})$ correspond to the codewords of \mathcal{C} . Two distinct vertices in $\Gamma(\mathcal{C})$ are adjacent if and only if the weight of the difference between their corresponding codewords is w_1 .

THEOREM 2.2 (Delsarte [3, Theorem 2]). *$\Gamma(\mathcal{C})$ is a strongly regular graph for any projective two-weight linear code \mathcal{C} .*

¹<http://linearcodes.uni-bayreuth.de/twoweight/>

3. The Segre embedding in $\text{PG}(5, q)$

Our motivation stemmed from the fact that many of the most interesting two-intersection sets of $\text{PG}(5, 3)$ had a stabiliser divisible by 7, and 7 is a primitive prime divisor of $3^6 - 1$ (and so an element of order 7 must act irreducibly). The Hill 56-cap and the examples found by Gulliver could be constructed by taking unions of orbits of this cyclic subgroup. Restricting the search to two-intersection sets that could be constructed in this manner prevented the scope of the problem from becoming infeasible and allowed for a full enumeration of possible solutions. To classify all two-weight ternary codes of dimension 6 seems to be out of the current range of computational power.

Let us model $\text{PG}(5, q)$ simply as the field extension $\mathbb{F}_{q^6}^\times$ over \mathbb{F}_q^\times . If ω is a primitive root of $\mathbb{F}_{q^6}^\times$, then multiplication by ω yields a cyclic subgroup acting regularly on the points of $\text{PG}(5, q)$; the so-called *Singer cycle* of $\text{PG}(5, q)$. If we raise ω to a suitable power, namely $(q+1)(q^2+q+1)$, then we will obtain a field element of order $(q-1)(q^2-q+1)$ in $\mathbb{F}_{q^6}^\times$, which will induce a collineation τ of $\text{PG}(5, q)$ of order $q^2 - q + 1$. Alternatively, we can write the orbits of τ down by taking the equivalence classes of the following relation:

$$(1) \quad x \sim y \Leftrightarrow x^{(q^2-q+1)(q-1)} = y^{(q^2-q+1)(q-1)},$$

where $x, y \in \mathbb{F}_{q^6}^\times$.

REMARK 3.1. *A two-intersection set invariant under τ will automatically span $\text{PG}(5, q)$ since τ acts irreducibly on this space.*

The map $\sigma : \text{PG}(1, q) \times \text{PG}(2, q) \rightarrow \text{PG}(5, q)$ defined by

$$\sigma([X_1, X_2], [Y_1, Y_2, Y_3]) = [X_1Y_1, X_1Y_2, X_1Y_3, X_2Y_1, X_2Y_2, X_2Y_3]$$

is a *Segre embedding*, and the image of this map is a cubic *Segre variety* $\mathcal{S}_{1,2}$ (see [9, Chapter 25]). In order to work with this Segre embedding, it became necessary to find an equivalent operation over elements of \mathbb{F}_{q^6} .

PROPOSITION 3.2. *The map $(x, y) \mapsto xy$ from $\mathbb{F}_{q^2}^\times \times \mathbb{F}_{q^3}^\times$ to $\mathbb{F}_{q^6}^\times$ is equivalent to the Segre embedding σ .*

PROOF. We model $\text{PG}(1, q)$ as \mathbb{F}_{q^2} over \mathbb{F}_q with basis $\{\chi_1, \chi_2\}$, and $\text{PG}(2, q)$ as \mathbb{F}_{q^3} over \mathbb{F}_q with basis $\{\psi_1, \psi_2, \psi_3\}$. Now suppose we have $x = [X_1, X_2] \in \text{PG}(1, q)$ and $y = [Y_1, Y_2, Y_3] \in \text{PG}(2, q)$; that is, $x = X_1\chi_1 + X_2\chi_2$ and $y = Y_1\psi_1 + Y_2\psi_2 + Y_3\psi_3$.

Clearly,

$$xy = X_1Y_1\chi_1\psi_1 + X_1Y_2\chi_1\psi_2 + X_1Y_3\chi_1\psi_3 + X_2Y_1\chi_2\psi_1 + X_2Y_2\chi_2\psi_2 + X_2Y_3\chi_2\psi_3,$$

so it suffices to show that $\{\chi_1\psi_1, \chi_1\psi_2, \chi_1\psi_3, \chi_2\psi_1, \chi_2\psi_2, \chi_2\psi_3\}$ is a basis for \mathbb{F}_{q^6} (over \mathbb{F}_q).

To this end, suppose $\lambda_1\chi_1\psi_1 + \lambda_2\chi_1\psi_2 + \lambda_3\chi_1\psi_3 + \lambda_4\chi_2\psi_1 + \lambda_5\chi_2\psi_2 + \lambda_6\chi_2\psi_3 = 0$. Further suppose, for the sake of contradiction, that $\lambda_1\psi_1 + \lambda_2\psi_2 + \lambda_3\psi_3 \neq 0$. We may then rearrange the equation as follows:

$$-\chi_1\chi_2^{-1} = (\lambda_1\psi_1 + \lambda_2\psi_2 + \lambda_3\psi_3)^{-1}(\lambda_4\psi_1 + \lambda_5\psi_2 + \lambda_6\psi_3).$$

Noting that the left hand side of the equation is an element of \mathbb{F}_{q^2} , and the right hand side of the equation is an element of \mathbb{F}_{q^3} , we conclude that both sides of the equation are elements of $\mathbb{F}_{q^2} \cap \mathbb{F}_{q^3} = \mathbb{F}_q$. Then we must have $\chi_1 = \lambda\chi_2$, for some $\lambda \in \mathbb{F}_q$ – a contradiction.

Hence, we must have $\lambda_1\psi_1 + \lambda_2\psi_2 + \lambda_3\psi_3 = 0$ and consequently, $\lambda_4\psi_1 + \lambda_5\psi_2 + \lambda_6\psi_3 = 0$. As ψ_1, ψ_2 and ψ_3 are linearly independent, we see that $\lambda_1 = \lambda_2 = \dots = \lambda_6 = 0$. This confirms that $\{\chi_1\psi_1, \chi_1\psi_2, \chi_1\psi_3, \chi_2\psi_1, \chi_2\psi_2, \chi_2\psi_3\}$ is a basis for \mathbb{F}_{q^6} . \square

It was discovered that, for certain values of q , the image of σ is a transversal of the orbits discussed above.

PROPOSITION 3.3. *For $q \not\equiv 2 \pmod{3}$, every equivalence class of (1) contains exactly one element of the Segre variety arising from the products $\mathcal{S} = \{xy : x \in \mathbb{F}_{q^2}^\times, y \in \mathbb{F}_{q^3}^\times\}$.*

PROOF. Consider the following subgroup of $\mathbb{F}_{q^6}^\times$ of order $(q^2 - q + 1)(q - 1)$:

$$\mathcal{W} := \{x \in \mathbb{F}_{q^6}^\times : x^{(q^2 - q + 1)(q - 1)} = 1\}.$$

We will first show that the subgroup $\mathcal{S} \cap \mathcal{W}$ is contained in \mathbb{F}_q^\times .

The order of the intersection $\mathcal{S} \cap \mathcal{W}$ must divide the order of both subgroups, and hence must also divide

$$\gcd((q - 1)(q + 1)(q^2 + q + 1), (q^2 - q + 1)(q - 1)) = (q - 1) \gcd((q + 1)(q^2 + q + 1), q^2 - q + 1).$$

Noting that $q^2 + q + 1$ and $q^2 - q + 1$ are coprime, we may simplify the expression to $(q - 1) \gcd(q + 1, q^2 - q + 1)$. This can be further simplified to $(q - 1) \gcd(q + 1, 3)$, with one step of the Euclidean algorithm. Clearly, when $q \not\equiv 2 \pmod{3}$, the above expression evaluates to $q - 1$. In these cases, the order of $\mathcal{S} \cap \mathcal{W}$ divides the order of \mathbb{F}_q^\times , and hence $\mathcal{S} \cap \mathcal{W}$ is a subgroup of \mathbb{F}_q^\times .

Now we may proceed to proving the stated result. Suppose $x, x' \in \mathbb{F}_{q^2}^\times$ and $y, y' \in \mathbb{F}_{q^3}^\times$. We show that if xy and $x'y'$ are in the same equivalence class of (1), they represent the same element of the Segre variety. To do so, we suppose that $(xy)^{(q^2 - q + 1)(q - 1)} = (x'y')^{(q^2 - q + 1)(q - 1)}$. This equation can be rearranged into the form $[(x'x^{-1})(y'y^{-1})]^{(q^2 - q + 1)(q - 1)} = 1$.

So $(x'x^{-1})(y'y^{-1})$ is an element of $\mathcal{S} \cap \mathcal{W}$, and is therefore in \mathbb{F}_q^\times , using the fact shown above. Hence, we have $(x'x^{-1})(y'y^{-1}) = \lambda$, for some $\lambda \in \mathbb{F}_q^\times$, which may be rearranged into the form $x'y' = \lambda xy$. Therefore, xy and $x'y'$ are projectively equivalent. We have shown, then, that the Segre variety arising from \mathcal{S} is a transversal of the given equivalence classes, as there are an equal number of equivalence classes and elements of \mathcal{S} . \square

This relationship between the Segre variety and the orbits of a collineation group allows for the expression of conforming two-intersection sets as geometric entities in the Segre variety. Every orbit included in such a two-intersection set corresponds to exactly one point in the Segre variety.

EXAMPLE 3.4. *Fix a primitive root ω of \mathbb{F}_{36} . Then we can describe the eight orbits comprising the Hill 56-cap by single elements of the Segre variety:*

$$1, \omega^{91}, \omega^{14}, \omega^{42}, \omega^{105}, \omega^{126}, \omega^{133}, \omega^{217}.$$

These elements of the Segre variety were obtained by taking the products $\omega^a \omega^b$, where

$$(a, b) \in \{(0, 0), (0, 91), (560, 182), (588, 182), (560, 273), (672, 182), (588, 273), (672, 273)\}.$$

Note that the first coordinates of these pairs are each divisible by 28, and the second coordinates are each divisible by 91 (and so $\omega^a \in \mathbb{F}_{q^3}$ and $\omega^b \in \mathbb{F}_{q^2}$ for each (a, b)). We will catalogue each of our examples in Section 7 by points of the Segre variety in this way.

4. Necessary conditions for the existence of two-intersection sets

The following is an extension of the results presented by Penttila and Royle [13, §2], and was used to quickly identify implausible parameter sets in the computational search.

PROPOSITION 4.1. *Suppose \mathcal{K} is a two-intersection set with parameters (n, k, h_1, h_2) . Then there must exist an integer solution to:*

$$(2) \quad n^2 \frac{q^{k-2} - 1}{q - 1} + n((1 - h_1 - h_2) \frac{q^{k-1} - 1}{q - 1} - \frac{q^{k-2} - 1}{q - 1}) + h_1 h_2 \frac{q^k - 1}{q - 1} = 0.$$

PROOF. If t_1 and t_2 are the number of hyperplanes that are incident with h_1 and h_2 points in \mathcal{K} , respectively, then counting arguments reveal the following equations:

$t_1 + t_2 = (q^k - 1)/(q - 1)$	counting the number of hyperplanes
$h_1 t_1 + h_2 t_2 = n(q^{k-1} - 1)(q - 1)$	counting incident point-hyperplane pairs
$h_1(h_1 - 1)t_1 + h_2(h_2 - 1)t_2 = n(n - 1)(q^{k-2} - 1)/(q - 1)$	counting triples (p_1, p_2, Π) where p_1 and p_2 are two different points incident with the hyperplane Π

If we take the left-hand sides of these three equations, and sum them with coefficients $h_1 h_2$, $1 - h_1 - h_2$, and 1 accordingly, we will obtain 0. The result then follows. \square

PROPOSITION 4.2. *Suppose \mathcal{K} is a two-intersection set with parameters (n, k, h_1, h_2) . Then $h_2 - h_1$ must divide q^{k-2} .*

PROOF. Suppose that ρ_1 and ρ_2 are the number of hyperplanes through a point $P \in \mathcal{K}$ that are incident with h_1 and h_2 points in \mathcal{K} , respectively. Similarly, suppose σ_1 and σ_2 are the number of hyperplanes through a point $Q \notin \mathcal{K}$ that are incident with h_1 and h_2 points in \mathcal{K} , respectively. Then counting arguments show that:

$$(3) \quad \rho_1 + \rho_2 = \frac{q^{k-1} - 1}{q - 1},$$

$$(4) \quad \sigma_1 + \sigma_2 = \frac{q^{k-1} - 1}{q - 1},$$

$$(5) \quad (h_1 - 1)\rho_1 + (h_2 - 1)\rho_2 = (n - 1)\frac{q^{k-2} - 1}{q - 1}, \text{ and}$$

$$(6) \quad h_1\sigma_1 + h_2\sigma_2 = n\frac{q^{k-2} - 1}{q - 1}.$$

By solving this system of equations, we see that $\rho_2 - \sigma_2 = \frac{q^{k-2}}{h_2 - h_1}$ must be an integer. \square

5. A computational search for two-intersection sets

A significant advantage of searching for two-intersection sets, rather than the other mathematical objects discussed in this paper, is the relative ease with which two-intersection sets can be found computationally. The task of finding two-intersection sets in a projective space can be naturally represented as a linear programming problem, as evinced by Kohnert [11]. This section details a method of framing the task in such a way.

Let G be a group of collineations of $\text{PG}(5, q)$. We let M be the G -quotient of the point-hyperplane incidence matrix wherein the $(i, j)^{\text{th}}$ entry is the number of elements from the i th G -orbit on points incident with the j th G -orbit representative on hyperplanes. Alternatively, we can use a duality arising from a bilinear form, say, so that the hyperplanes are replaced by points and incidence is replaced by orthogonality. In our application, where G is a cyclic group of order $q^2 - q + 1$ acting on $\text{PG}(5, q)$, we can simply construct M by indexing its rows and columns by the cubic Segre variety $\mathcal{S}_{1,2}$. The relative trace map (from \mathbb{F}_{q^6} to \mathbb{F}_q) yields a bilinear form when we model $\text{PG}(5, q)$ by the field \mathbb{F}_{q^6} :

$$B(x, y) := \text{Tr}_{q^6 \rightarrow q}(xy).$$

Hence, we may instead calculate $M(i, j)$ by calculating the number of elements $g \in G$ such that $B(s_i g, s_j) = 0$, where s_i, s_j are the i -th and j -th elements of the Segre variety modelled in \mathbb{F}_{q^6} .

An *orbit inclusion vector* is a 0-1 vector in which the i th entry is 1 or 0, dependent on whether the i -th orbit is included or excluded from the current point set (resp.). Clearly, premultiplying the inclusion vector with the incidence matrix yields a vector wherein the i -th entry contains the number of points in the current point set incident with the i -th hyperplane.

In order to find a two-intersection set with intersection values h_1 and h_2 , we need to find solutions to the system of equations:

$$\mathbf{x}M = (h_1 \text{ or } h_2, \quad h_1 \text{ or } h_2, \quad \dots, \quad h_1 \text{ or } h_2),$$

where \mathbf{x} is a 0-1 vector. While this is not a linear system, it can be converted into one with a minor modification. For a given solution \mathbf{x} , we introduce a vector \mathbf{y} of length n , such that the i th entry of \mathbf{y} is 0 if the i th hyperplane is incident with h_1 points, or 1 if the i th hyperplane is incident with h_2 points. This allows the problem to be framed as a linear system of equations:

$$\mathbf{x}M + (h_1 - h_2)\mathbf{y} = (h_1, h_1, \dots, h_1),$$

which can then be solved with software such as *Gurobi* [10]. This also has the benefit of immediately revealing which hyperplanes are incident with h_1 points and which are incident with h_2 .

6. Optimising the search

An initial search for two-intersection sets, filtered only by fulfilment of the necessary conditions discussed above, was able to return hundreds of thousands of results without exhausting the problem space. To prevent the re-computation of redundant data, the action of the full collineation group was used to impose a number of constraints on the search. We computed valuable information regarding the symmetries of the projective space using the free software package GAP [4]. Firstly, it was noted that the normaliser of our element τ of order 7 in the full collineation group acted transitively on the point-orbits of τ , meaning any solution was equivalent to one that included the first orbit. Hence, the first orbit was selected in every one of the reported solutions.

Next, the subgroup that stabilised the first orbit was examined. This group had nine orbits itself, represented by the orbits with indices in the set $\{1, 2, 3, 5, 6, 17, 18, 20, 24\}$. Hence, any solution that included two or more orbits was equivalent to one that included either orbits 1 and 2, or orbits 1 and 3, or orbits 1 and 5 etc. The solution space was then restricted to ensure that one of these orbit pairs was always included.

This method was reapplied to those subproblems that proved computationally difficult. For instance, including orbits 1 and 2 in a solution was shown to be equivalent to including one of nine triples of orbits. In this manner, a tree of computations was formed and traversed, sometimes reaching seven levels deep.

One optimisation that proved particularly efficient was facilitated by the use of the branching method discussed above. As noted, there are many sets of orbits that are equivalent to one another. The search was accelerated by precluding the consideration of any set equivalent to one already encountered. For example, there exist 52 pairs of orbits whose inclusion is equivalent to the inclusion of the orbits 1 and 2. Hence, after all solutions containing orbits 1 and 2 have been found, any of these 52 equivalent pairs of orbits can be excluded from appearing again in any future results.

These optimisation techniques allowed for the enumeration of all τ -invariant two-intersection sets in $\text{PG}(5, 3)$ within a viable time-frame (approximately 2 weeks).

In order to have confidence in the correctness of the search, we used the constraint satisfaction solver MINION [5] to simulate some of our results. In particular, we enumerated all the two-intersection sets with parameters $\{11, 20\}$, $\{21, 30\}$, $\{26, 35\}$, $\{28, 37\}$, or $\{31, 40\}$, invariant under a group with order divisible by 7, and we obtained the same results which we outline in Table 1:

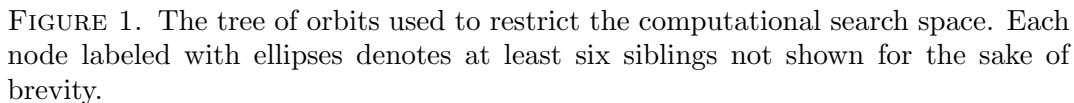


TABLE 1. Examples for small parameter sets.

7. Results

²Sylvia Morris, ‘Symplectic translation planes, pseudo-ovals, and maximal 4-arcs’, dissertation for the Master of Philosophy (Research) of The University of Western Australia (2014).

given two-intersection set, in the following format: each entry specifies a pair of exponents to the primitive root ω of \mathbb{F}_{36} , which is represented as $\mathbb{F}_3[x]/\langle x^6 - x^4 + x^2 - x - 1 \rangle$. The specific element of the Segre variety can be obtained by applying the multiplication map to the resulting elements. That is, the map $(a, b) \mapsto \omega^a \omega^b$ yields the correct element of the Segre variety in the model \mathbb{F}_{36}^\times over \mathbb{F}_3^\times . The “stabiliser size” column lists the size of the largest group that stabilises the given orbit set. It is important to note that not all two-intersection sets, and hence not all strongly regular graphs, are present in the tables that follow: 6635 solutions were found altogether, with all but the 187 listed below having a full stabiliser of order 7 in the collineation group. Every one of the 6635 computed two-intersection sets was found to correspond to a unique strongly regular graph. This was achieved by using the software **nauty** [12] to identify the canonical representation of the graph. The full list of two-intersection sets can be found at the webpage <https://researchdataonline.research.uwa.edu.au/handle/123456789/1358>.

Table 2: Two-intersection sets of type (56, 6, 11, 20)

Segre variety representation	Stabiliser size
(0, 0), (0, 91), (560, 182), (588, 182), (560, 273), (672, 182), (588, 273), (672, 273)	40320

The only two-intersection set of size 56 is the Hill cap [8], which gives rise to a strongly regular graph with parameters (729, 112, 1, 20) (this is also Example FE2 in [2, §11]).

Table 3: Two-intersection sets of type (84, 6, 21, 30)

Segre variety representation	Stabiliser size
(0, 0), (0, 182), (168, 0), (588, 364), (672, 91), (588, 182), (28, 91), (56, 91), (448, 455), (476, 455), (560, 455), (168, 182)	42
(0, 0), (0, 182), (504, 364), (560, 364), (560, 182), (588, 273), (672, 273), (700, 273), (28, 273), (504, 546), (56, 273), (448, 637)	42

The above two-intersection sets give rise to strongly regular graphs with parameters (729, 168, 27, 42). Gulliver’s (84, 6, 54)-code [7] has a stabiliser of order 7 so does not appear in this list.

Table 4: Two-intersection sets of type (98, 6, 26, 35)

Segre variety representation	Stabiliser size
(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (504, 364), (168, 0), (252, 0), (252, 546), (560, 273), (588, 273), (28, 182), (672, 273), (448, 546)	21
(0, 0), (28, 0), (504, 364), (672, 364), (168, 637), (560, 273), (28, 91), (588, 273), (252, 637), (448, 455), (700, 273), (168, 91), (560, 455), (448, 637)	21

The above two-intersection sets give rise to strongly regular graphs with parameters (729, 196, 43, 56). Gulliver’s (98, 6, 63)-code [7] has a stabiliser of order 7 so does not appear in this list.

Table 5: Two-intersection sets of type (91, 6, 28, 37)

Segre variety representation	Stabiliser size
(0, 0), (0, 91), (0, 182), (0, 273), (56, 0), (252, 0), (672, 364), (700, 364), (588, 182), (252, 546), (672, 182), (448, 455), (448, 637)	14
(0, 0), (0, 91), (0, 273), (504, 364), (560, 364), (504, 273), (560, 273), (476, 455), (672, 273), (504, 455), (700, 273), (560, 455), (56, 273)	168
(0, 0), (0, 91), (0, 182), (0, 273), (476, 364), (280, 0), (700, 364), (280, 546), (28, 91), (700, 182), (448, 455), (476, 546), (252, 91)	42*

Table 5: Two-intersection sets of type (91, 6, 28, 37)

Segre variety representation	Stabiliser size
(0, 0), (28, 0), (280, 0), (560, 182), (476, 273), (588, 273), (56, 91), (700, 182), (252, 637), (448, 455), (672, 273), (168, 91), (504, 546)	21
(0, 0), (28, 0), (56, 0), (448, 364), (504, 364), (168, 0), (560, 364), (588, 364), (280, 0), (700, 364), (252, 546), (672, 182), (476, 546)	42
(0, 0), (28, 0), (56, 0), (448, 364), (168, 0), (280, 0), (560, 182), (588, 182), (252, 546), (672, 182), (700, 182), (476, 546), (504, 546)	42
(0, 0), (28, 0), (56, 0), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (280, 0), (252, 546), (672, 182), (700, 182), (448, 546)	42
(0, 0), (28, 0), (448, 364), (168, 0), (252, 0), (672, 364), (560, 182), (588, 182), (280, 546), (700, 182), (56, 182), (476, 546), (504, 546)	14
(0, 0), (28, 0), (168, 0), (252, 0), (280, 0), (700, 364), (560, 182), (588, 182), (672, 182), (56, 182), (448, 546), (476, 546), (504, 546)	42
(0, 0), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (560, 364), (252, 0), (280, 0), (588, 182), (672, 182), (700, 182), (168, 182)	42
(0, 0), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (700, 364)	546**
(0, 0), (28, 0), (280, 0), (560, 182), (476, 273), (168, 637), (700, 182), (252, 637), (672, 273), (588, 455), (504, 546), (56, 273), (448, 637)	21*
(0, 0), (28, 0), (56, 0), (476, 364), (504, 364), (588, 364), (252, 0), (280, 0), (560, 182), (672, 182), (700, 182), (448, 546), (168, 182)	14

The above two-intersection sets give rise to strongly regular graphs with parameters (729, 182, 55, 42). The three examples in the table above that have an asterisk in the last column arise from partial spreads of $\text{PG}(5, 3)$; the ‘SU2’ construction in [2, §7]. One of these examples also arises from the ‘CY1’ construction in [2, §9], denoted by two asterisks.

Table 6: Two-intersection sets of type (112, 6, 31, 40)

Segre variety representation	Stabiliser size
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (476, 364), (280, 455), (476, 273), (252, 546), (280, 546), (168, 91), (28, 273), (56, 273), (252, 91), (168, 182)	14
(0, 0), (0, 91), (0, 182), (0, 273), (56, 0), (280, 0), (252, 546), (28, 91), (280, 637), (476, 455), (28, 182), (168, 91), (476, 546), (56, 273), (252, 91), (168, 182)	14
(0, 0), (0, 91), (0, 182), (0, 273), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (560, 273), (588, 273), (56, 91), (252, 637), (280, 637), (672, 273), (56, 182)	14
(0, 0), (0, 91), (0, 182), (0, 273), (476, 364), (560, 364), (252, 0), (560, 182), (504, 273), (700, 91), (252, 546), (252, 637), (672, 273), (504, 455), (168, 91), (476, 546)	14
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (476, 364), (168, 0), (476, 273), (168, 637), (28, 91), (476, 455), (28, 182), (168, 91), (476, 546), (28, 273), (168, 182)	26127360
(0, 0), (0, 91), (0, 182), (56, 0), (168, 0), (588, 364), (280, 455), (476, 273), (588, 182), (28, 91), (252, 637), (448, 455), (672, 273), (56, 182), (560, 455), (168, 182)	14
(0, 0), (0, 91), (0, 182), (504, 364), (168, 0), (672, 364), (700, 91), (28, 91), (672, 182), (448, 455), (476, 455), (560, 455), (588, 455), (504, 546), (252, 91), (168, 182)	14
(0, 0), (0, 182), (504, 364), (560, 364), (672, 364), (560, 182), (700, 91), (168, 637), (672, 182), (588, 273), (56, 91), (448, 455), (504, 455), (28, 273), (504, 546), (252, 91)	14
(0, 0), (0, 182), (56, 0), (588, 364), (280, 0), (476, 273), (588, 182), (168, 637), (280, 546), (560, 273), (448, 455), (672, 273), (504, 455), (56, 182), (588, 455), (252, 91)	14

The above two-intersection sets give rise to strongly regular graphs with parameters (729, 224, 61, 72). The example above with stabiliser of order 26127360 is none-other than the points of the elliptic quadric $Q^-(5, 3)$ (this is also Example RT2 in [2, §10]).

Table 7: Two-intersection sets of type $(126, 6, 36, 45)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 182), (28, 0), (504, 364), (560, 364), (280, 0), (700, 364), (560, 182), (476, 273), (252, 546), (280, 546), (672, 182), (700, 182), (252, 637), (28, 182), (672, 273), (476, 546), (504, 546)$	21
$(0, 0), (0, 273), (28, 0), (56, 0), (476, 364), (504, 364), (588, 364), (252, 546), (28, 91), (672, 182), (588, 273), (252, 637), (476, 455), (672, 273), (504, 455), (448, 546), (56, 273), (448, 637)$	21
$(0, 0), (0, 182), (28, 0), (448, 364), (476, 364), (560, 364), (700, 364), (280, 455), (560, 182), (588, 273), (56, 91), (700, 182), (280, 637), (28, 182), (448, 546), (476, 546), (588, 455), (56, 273)$	13063680
$(0, 0), (0, 273), (28, 0), (672, 364), (700, 364), (280, 455), (588, 182), (280, 546), (56, 91), (672, 273), (504, 455), (56, 182), (700, 273), (168, 91), (28, 273), (588, 455), (504, 546), (168, 182)$	14
$(0, 0), (0, 91), (28, 0), (448, 364), (476, 364), (560, 364), (700, 364), (588, 182), (700, 91), (280, 546), (28, 91), (588, 273), (448, 455), (280, 637), (476, 455), (56, 182), (560, 455), (56, 273)$	42
$(0, 0), (0, 91), (448, 364), (476, 364), (560, 182), (588, 182), (280, 546), (560, 273), (588, 273), (700, 182), (448, 455), (280, 637), (476, 455), (28, 182), (56, 182), (700, 273), (28, 273), (56, 273)$	42
$(0, 0), (0, 91), (0, 182), (56, 0), (448, 364), (476, 364), (588, 364), (280, 0), (588, 182), (280, 546), (588, 273), (448, 455), (280, 637), (476, 455), (56, 182), (448, 546), (476, 546), (56, 273)$	42
$(0, 0), (0, 91), (0, 182), (0, 273), (504, 364), (560, 364), (560, 182), (504, 273), (168, 637), (560, 273), (28, 91), (252, 637), (448, 455), (280, 637), (504, 455), (560, 455), (588, 455), (504, 546)$	42
$(0, 0), (0, 91), (476, 364), (504, 364), (168, 0), (252, 0), (560, 182), (588, 182), (560, 273), (588, 273), (700, 182), (476, 455), (504, 455), (56, 182), (700, 273), (168, 91), (56, 273), (252, 91)$	14

The above two-intersection sets give rise to strongly regular graphs with parameters $(729, 252, 81, 90)$. The example above with stabiliser of order 13063680 can be constructed by taking a quadratic form Q of minus type defining an elliptic quadric $Q^-(5, 3)$, and then considering the 126 points p such that $Q(p) = 1$.

Table 8: Two-intersecetion sets of type $(140, 6, 41, 50)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 91), (0, 182), (28, 0), (476, 364), (280, 455), (588, 182), (504, 273), (168, 637), (28, 91), (672, 182), (700, 182), (448, 455), (280, 637), (476, 455), (28, 182), (504, 455), (476, 546), (168, 182), (448, 637)$	21
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (588, 364), (672, 364), (700, 91), (28, 91), (588, 273), (56, 91), (700, 182), (448, 455), (28, 182), (672, 273), (56, 182), (448, 546), (28, 273), (56, 273)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (476, 364), (168, 637), (280, 546), (560, 273), (28, 91), (56, 91), (252, 637), (476, 455), (28, 182), (448, 546), (560, 455), (476, 546), (504, 546), (56, 273), (252, 91), (168, 182)$	21
$(0, 0), (0, 91), (0, 182), (28, 0), (280, 0), (280, 455), (560, 182), (672, 91), (588, 182), (252, 546), (280, 546), (28, 91), (672, 182), (700, 182), (476, 455), (28, 182), (476, 546), (588, 455), (504, 546), (252, 91)$	21
$(0, 0), (0, 91), (0, 182), (28, 0), (504, 364), (560, 364), (588, 364), (280, 0), (700, 364), (280, 455), (476, 273), (252, 546), (280, 546), (28, 91), (672, 182), (252, 637), (28, 182), (672, 273), (476, 546), (588, 455)$	21
$(0, 0), (0, 91), (0, 182), (28, 0), (476, 364), (504, 364), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (700, 364), (280, 455), (476, 273), (280, 546), (28, 91), (252, 637), (28, 182), (672, 273), (588, 455)$	21
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (476, 364), (504, 364), (588, 364), (700, 364), (588, 182), (700, 91), (28, 91), (588, 273), (56, 91), (476, 455), (504, 455), (56, 182), (588, 455), (56, 273)$	42
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (476, 364), (504, 364), (280, 455), (560, 182), (588, 182), (700, 91), (28, 91), (588, 273), (280, 637), (56, 182), (700, 273), (28, 273), (588, 455), (504, 546)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (588, 364), (280, 455), (588, 182), (252, 637), (476, 455), (28, 182), (672, 273), (56, 182), (700, 273), (560, 455), (588, 455), (252, 91), (168, 182), (448, 637)$	14

Table 8: Two-intersection sets of type $(140, 6, 41, 50)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 91), (0, 182), (28, 0), (476, 364), (252, 0), (280, 0), (672, 364), (280, 455), (560, 182), (672, 91), (588, 182), (280, 546), (28, 91), (700, 182), (476, 455), (28, 182), (588, 455), (504, 546), (252, 91)$	21
$(0, 0), (0, 91), (0, 182), (56, 0), (504, 364), (588, 364), (672, 364), (588, 182), (168, 637), (672, 182), (252, 637), (448, 455), (280, 637), (56, 182), (700, 273), (168, 91), (560, 455), (504, 546), (252, 91), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (56, 0), (504, 364), (560, 364), (672, 364), (280, 455), (560, 182), (504, 273), (700, 91), (28, 91), (672, 182), (588, 273), (252, 637), (56, 182), (168, 91), (504, 546), (252, 91), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (56, 0), (504, 364), (560, 364), (672, 364), (560, 182), (504, 273), (672, 182), (252, 637), (280, 637), (56, 182), (700, 273), (168, 91), (28, 273), (588, 455), (504, 546), (252, 91), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (56, 0), (476, 364), (168, 0), (588, 364), (672, 91), (588, 182), (560, 273), (28, 91), (56, 91), (252, 637), (448, 455), (280, 637), (672, 273), (56, 182), (700, 273), (476, 546), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (56, 0), (168, 0), (588, 364), (280, 0), (476, 273), (588, 182), (280, 546), (28, 91), (252, 637), (448, 455), (476, 455), (672, 273), (504, 455), (56, 182), (560, 455), (588, 455), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (56, 0), (504, 364), (588, 364), (280, 0), (476, 273), (588, 182), (280, 546), (588, 273), (448, 455), (672, 273), (56, 182), (700, 273), (168, 91), (560, 455), (504, 546), (252, 91), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (56, 0), (504, 364), (560, 364), (672, 364), (560, 182), (168, 637), (28, 91), (672, 182), (252, 637), (280, 637), (504, 455), (56, 182), (700, 273), (588, 455), (504, 546), (252, 91), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (56, 0), (476, 364), (168, 0), (672, 364), (504, 273), (700, 91), (672, 182), (588, 273), (252, 637), (280, 637), (672, 273), (56, 182), (560, 455), (476, 546), (28, 273), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (56, 0), (504, 364), (560, 364), (280, 0), (560, 182), (476, 273), (672, 91), (700, 91), (280, 546), (588, 273), (504, 455), (56, 182), (28, 273), (588, 455), (504, 546), (252, 91), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (504, 364), (560, 364), (588, 364), (280, 0), (700, 364), (280, 455), (504, 273), (280, 546), (560, 273), (28, 91), (588, 273), (280, 637), (28, 182), (700, 273), (28, 273)$	42
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (588, 364), (588, 182), (560, 273), (252, 637), (448, 455), (280, 637), (476, 455), (28, 182), (672, 273), (56, 182), (700, 273), (588, 455), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (588, 364), (280, 455), (476, 273), (588, 182), (700, 91), (252, 637), (448, 455), (28, 182), (672, 273), (56, 182), (560, 455), (588, 455), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (588, 364), (476, 273), (588, 182), (700, 91), (588, 273), (252, 637), (280, 637), (28, 182), (672, 273), (56, 182), (560, 455), (252, 91), (168, 182), (448, 637)$	14

The above two-intersection sets give rise to strongly regular graphs with parameters $(729, 280, 103, 110)$.

Table 9: Two-intersection sets of type $(154, 6, 46, 55)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (560, 182), (476, 273), (672, 91), (588, 182), (700, 91), (560, 273), (252, 637), (476, 455), (28, 182), (672, 273), (56, 182), (168, 91), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (448, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (672, 364), (672, 91), (252, 546), (560, 273), (588, 273), (28, 182), (672, 273), (448, 546), (560, 455), (588, 455)$	42

Table 9: Two-intersection sets of type $(154, 6, 46, 55)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (588, 364), (252, 0), (560, 182), (588, 182), (168, 637), (560, 273), (28, 91), (588, 273), (56, 91), (28, 182), (56, 182), (28, 273), (588, 455), (56, 273), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (168, 0), (252, 0), (672, 91), (700, 91), (252, 546), (560, 273), (28, 91), (56, 91), (448, 455), (28, 182), (672, 273), (504, 455), (56, 182), (28, 273), (56, 273), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (448, 364), (504, 364), (560, 364), (280, 0), (700, 364), (672, 91), (700, 91), (280, 546), (28, 91), (28, 182), (672, 273), (700, 273), (448, 546), (476, 546), (28, 273), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (476, 364), (252, 0), (280, 0), (280, 455), (672, 91), (252, 546), (168, 637), (280, 546), (560, 273), (56, 91), (280, 637), (476, 455), (28, 182), (56, 182), (560, 455), (476, 546)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (476, 364), (588, 364), (700, 364), (280, 455), (588, 182), (28, 91), (56, 91), (700, 182), (280, 637), (28, 182), (56, 182), (476, 546), (28, 273), (588, 455), (252, 91), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (448, 364), (504, 364), (168, 0), (252, 0), (504, 273), (168, 637), (56, 91), (252, 637), (504, 455), (56, 182), (168, 91), (28, 273), (504, 546), (56, 273), (168, 182), (448, 637)$	42
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (560, 364), (588, 364), (560, 182), (476, 273), (672, 91), (588, 182), (504, 273), (168, 637), (280, 546), (700, 182), (476, 455), (672, 273), (504, 455), (56, 182), (168, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (588, 364), (672, 364), (672, 91), (588, 182), (560, 273), (672, 182), (448, 455), (280, 637), (28, 182), (672, 273), (56, 182), (700, 273), (560, 455), (28, 273), (588, 455), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (476, 364), (504, 364), (560, 364), (588, 364), (280, 0), (280, 455), (560, 182), (672, 91), (588, 182), (280, 546), (560, 273), (476, 455), (28, 182), (168, 91), (476, 546), (504, 546), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (56, 0), (504, 364), (168, 0), (560, 364), (588, 364), (672, 364), (560, 182), (476, 273), (588, 182), (560, 273), (672, 182), (588, 273), (280, 637), (672, 273), (56, 182), (700, 273), (504, 546), (168, 182)$	42

The above two-intersection sets give rise to strongly regular graphs with parameters $(729, 308, 127, 132)$. These examples are different to Gulliver's $(154, 6, 99)$ -code [6], since his example has a stabiliser of order 11.

Table 10: Two-intersection sets of type $(168, 6, 51, 60)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (168, 0), (560, 364), (588, 364), (672, 364), (476, 273), (672, 91), (280, 546), (28, 91), (672, 182), (280, 637), (28, 182), (672, 273), (168, 91), (448, 546), (560, 455), (476, 546), (28, 273), (588, 455), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (560, 364), (588, 364), (672, 364), (672, 91), (588, 182), (168, 637), (28, 91), (588, 273), (56, 91), (448, 455), (476, 455), (168, 91), (560, 455), (588, 455), (168, 182)$	42
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (168, 0), (560, 364), (252, 0), (672, 364), (700, 364), (560, 182), (504, 273), (252, 546), (672, 182), (588, 273), (700, 182), (252, 637), (476, 455), (28, 182), (448, 546), (56, 273), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (476, 364), (168, 0), (560, 364), (588, 364), (280, 0), (700, 364), (560, 182), (476, 273), (672, 91), (168, 637), (560, 273), (672, 182), (588, 273), (280, 637), (700, 273), (168, 91), (560, 455), (28, 273), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (476, 364), (168, 0), (560, 364), (280, 0), (280, 455), (476, 273), (700, 91), (252, 546), (168, 637), (280, 546), (560, 273), (700, 182), (280, 637), (168, 91), (28, 273), (56, 273), (252, 91), (168, 182)$	14

Segre variety representation	Stabiliser size
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (672, 364), (560, 182), (476, 273), (588, 182), (700, 91), (252, 546), (56, 91), (700, 182), (476, 455), (28, 182), (700, 273), (504, 546), (56, 273)	14
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (672, 364), (672, 91), (168, 637), (28, 91), (672, 182), (476, 455), (672, 273), (504, 455), (168, 91), (560, 455), (588, 455), (252, 91), (168, 182)	42
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (476, 364), (168, 0), (588, 364), (252, 0), (280, 0), (700, 364), (588, 182), (252, 546), (280, 546), (560, 273), (56, 91), (700, 182), (28, 182), (672, 273), (168, 91), (560, 455), (476, 546), (28, 273), (168, 182)	14
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (476, 364), (168, 0), (560, 364), (252, 0), (280, 0), (560, 182), (672, 91), (588, 182), (504, 273), (252, 546), (280, 546), (672, 182), (588, 273), (672, 273), (504, 455), (476, 546), (588, 455), (168, 182)	14
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (504, 364), (168, 0), (252, 0), (280, 0), (560, 182), (504, 273), (168, 637), (28, 91), (56, 91), (252, 637), (280, 637), (476, 455), (28, 182), (56, 182), (560, 455), (476, 546), (28, 273), (56, 273)	14
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (168, 0), (280, 0), (672, 364), (280, 455), (560, 182), (476, 273), (672, 91), (588, 182), (560, 273), (28, 91), (672, 182), (588, 273), (28, 182), (672, 273), (168, 91), (448, 546), (476, 546), (28, 273), (448, 637)	14
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (476, 364), (168, 0), (252, 0), (672, 364), (700, 364), (280, 455), (700, 91), (560, 273), (588, 273), (56, 91), (700, 182), (280, 637), (28, 182), (700, 273), (448, 546), (560, 455), (476, 546), (588, 455), (56, 273)	14
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (168, 0), (280, 455), (560, 182), (504, 273), (700, 91), (560, 273), (28, 91), (588, 273), (252, 637), (280, 637), (504, 455), (56, 182), (700, 273), (560, 455), (28, 273), (588, 455), (252, 91), (168, 182)	14
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (476, 364), (560, 364), (588, 364), (252, 0), (700, 364), (280, 455), (504, 273), (168, 637), (168, 0), (280, 637), (672, 182), (588, 273), (56, 91), (280, 637), (504, 455), (168, 91), (448, 546), (476, 546), (588, 455), (56, 273)	14
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (560, 364), (280, 0), (700, 364), (560, 182), (476, 273), (672, 91), (168, 637), (280, 546), (588, 273), (56, 91), (700, 182), (280, 637), (28, 182), (168, 91), (448, 546), (588, 455), (56, 273), (448, 637)	14
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (560, 364), (280, 0), (588, 182), (504, 273), (700, 91), (252, 546), (168, 637), (280, 546), (560, 273), (672, 182), (56, 91), (28, 182), (504, 455), (700, 273), (168, 91), (560, 455), (476, 546), (56, 273)	14
(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (504, 364), (588, 364), (252, 0), (672, 91), (504, 273), (252, 546), (168, 637), (280, 546), (560, 273), (56, 91), (700, 182), (28, 182), (672, 273), (504, 455), (168, 91), (448, 546), (560, 455), (476, 546), (56, 273)	14
(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (560, 182), (588, 182), (504, 273), (700, 91), (588, 273), (56, 91), (252, 637), (448, 455), (280, 637), (476, 455), (28, 182), (672, 273), (56, 182), (28, 273), (252, 91), (168, 182)	14
(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (560, 182), (588, 182), (504, 273), (252, 637), (448, 455), (280, 637), (476, 455), (28, 182), (672, 273), (56, 182), (28, 273), (252, 91), (168, 182)	14
(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (560, 182), (588, 182), (504, 273), (252, 637), (448, 455), (280, 637), (476, 455), (28, 182), (672, 273), (56, 182), (28, 273), (252, 91), (168, 182)	14
(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (280, 455), (560, 182), (588, 182), (700, 91), (588, 273), (252, 637), (280, 637), (476, 455), (28, 182), (672, 273), (504, 455), (56, 182), (28, 273), (56, 273), (252, 91), (168, 182), (448, 637)	14
(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (280, 455), (560, 182), (588, 182), (700, 91), (252, 637), (448, 455), (476, 455), (28, 182), (672, 273), (504, 455), (56, 182), (28, 273), (588, 455), (56, 273), (252, 91), (168, 182)	14
(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (280, 455), (560, 182), (588, 182), (700, 91), (588, 273), (252, 637), (448,	

Table 10: Two-intersection sets of type $(168, 6, 51, 60)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (280, 0), (672, 364), (700, 364), (588, 182), (504, 273), (448, 455), (280, 637), (476, 455), (700, 273), (168, 91), (560, 455), (588, 455), (168, 182)$	21
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (560, 182), (476, 273), (672, 91), (588, 182), (700, 91), (252, 637), (280, 637), (28, 182), (504, 455), (56, 182), (28, 273), (588, 455), (56, 273), (252, 91), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (700, 364), (280, 455), (560, 182), (672, 91), (700, 91), (28, 91), (56, 91), (476, 455), (56, 182), (168, 91), (560, 455)$	21
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (560, 364), (588, 364), (672, 364), (588, 182), (504, 273), (280, 546), (700, 182), (448, 455), (280, 637), (476, 455), (700, 273), (168, 91), (560, 455), (588, 455), (504, 546), (168, 182)$	21
$(0, 0), (0, 91), (0, 182), (28, 0), (504, 364), (168, 0), (560, 364), (588, 364), (560, 182), (588, 182), (700, 91), (588, 273), (56, 91), (448, 455), (476, 455), (28, 182), (672, 273), (700, 273), (28, 273), (504, 546), (56, 273), (252, 91), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (560, 182), (476, 273), (672, 91), (588, 182), (504, 273), (700, 91), (56, 91), (252, 637), (448, 455), (280, 637), (28, 182), (56, 182), (28, 273), (588, 455), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (560, 182), (476, 273), (672, 91), (588, 182), (504, 273), (700, 91), (28, 91), (588, 273), (252, 637), (280, 637), (28, 182), (56, 182), (56, 273), (252, 91), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (504, 364), (168, 0), (560, 364), (588, 364), (560, 182), (476, 273), (672, 91), (588, 182), (700, 91), (56, 91), (448, 455), (28, 182), (700, 273), (28, 273), (588, 455), (504, 546), (56, 273), (252, 91), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (560, 364), (588, 364), (672, 364), (280, 455), (588, 182), (700, 91), (280, 546), (700, 182), (448, 455), (476, 455), (504, 455), (168, 91), (560, 455), (588, 455), (504, 546), (168, 182)$	21
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (280, 455), (560, 182), (476, 273), (588, 182), (700, 91), (28, 91), (56, 91), (252, 637), (448, 455), (28, 182), (672, 273), (504, 455), (56, 182), (588, 455), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (280, 455), (560, 182), (588, 182), (504, 273), (700, 91), (28, 91), (588, 273), (252, 637), (448, 455), (476, 455), (28, 182), (672, 273), (56, 182), (56, 273), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (280, 455), (560, 182), (476, 273), (672, 91), (588, 182), (504, 273), (700, 91), (28, 91), (252, 637), (448, 455), (28, 182), (56, 182), (588, 455), (56, 273), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (560, 182), (476, 273), (588, 182), (504, 273), (28, 91), (56, 91), (252, 637), (448, 455), (280, 637), (28, 182), (672, 273), (56, 182), (700, 273), (588, 455), (252, 91), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (560, 182), (476, 273), (588, 182), (504, 273), (28, 91), (56, 91), (252, 637), (448, 455), (280, 637), (28, 182), (672, 273), (56, 182), (700, 273), (588, 455), (252, 91), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (280, 455), (560, 182), (672, 91), (588, 182), (504, 273), (700, 91), (28, 91), (588, 273), (56, 91), (252, 637), (476, 455), (28, 182), (56, 182), (252, 91), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (560, 182), (476, 273), (588, 182), (700, 91), (28, 91), (588, 273), (56, 91), (252, 637), (280, 637), (28, 182), (672, 273), (504, 455), (56, 182), (252, 91), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (280, 455), (560, 182), (672, 91), (588, 182), (504, 273), (28, 91), (252, 637), (476, 455), (28, 182), (56, 182), (700, 273), (588, 455), (56, 273), (252, 91), (168, 182), (448, 637)$	14

Table 10: Two-intersection sets of type $(168, 6, 51, 60)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (560, 182), (476, 273), (588, 182), (28, 91), (252, 637), (280, 637), (28, 182), (672, 273), (504, 455), (56, 182), (700, 273), (588, 455), (56, 273), (252, 91), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (700, 364), (280, 455), (560, 182), (504, 273), (588, 273), (700, 182), (252, 637), (280, 637), (476, 455), (28, 182), (672, 273), (56, 182), (560, 455), (28, 273), (588, 455), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (560, 364), (700, 364), (280, 455), (560, 182), (672, 91), (504, 273), (560, 273), (28, 91), (588, 273), (700, 182), (252, 637), (280, 637), (476, 455), (28, 182), (56, 182), (588, 455), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (476, 364), (168, 0), (588, 364), (280, 455), (672, 91), (588, 182), (700, 91), (56, 91), (252, 637), (28, 182), (672, 273), (56, 182), (700, 273), (560, 455), (476, 546), (588, 455), (252, 91), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (168, 0), (588, 364), (700, 364), (280, 455), (588, 182), (168, 637), (560, 273), (700, 182), (252, 637), (280, 637), (476, 455), (28, 182), (672, 273), (56, 182), (560, 455), (588, 455), (252, 91), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (476, 364), (168, 0), (588, 364), (672, 91), (588, 182), (700, 91), (560, 273), (56, 91), (252, 637), (448, 455), (280, 637), (28, 182), (672, 273), (56, 182), (700, 273), (476, 546), (588, 455), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (448, 364), (168, 0), (588, 364), (280, 455), (672, 91), (588, 182), (560, 273), (28, 91), (252, 637), (280, 637), (476, 455), (28, 182), (672, 273), (56, 182), (700, 273), (448, 546), (588, 455), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (56, 0), (476, 364), (168, 0), (560, 364), (700, 364), (280, 455), (560, 182), (672, 91), (504, 273), (28, 91), (588, 273), (56, 91), (700, 182), (252, 637), (280, 637), (672, 273), (56, 182), (560, 455), (476, 546), (28, 273), (168, 182)$	42
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (476, 364), (504, 364), (168, 0), (560, 364), (252, 0), (560, 182), (588, 182), (700, 91), (252, 546), (588, 273), (56, 91), (28, 182), (700, 273), (448, 546), (588, 455), (504, 546), (56, 273), (168, 182)$	14

The above two-intersection sets give rise to strongly regular graphs with parameters $(729, 336, 153, 156)$.

Table 11: Two-intersection sets of type $(182, 6, 56, 65)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (168, 0), (560, 364), (476, 273), (252, 546), (168, 637), (28, 91), (672, 182), (700, 182), (252, 637), (448, 455), (28, 182), (672, 273), (56, 182), (700, 273), (168, 91), (560, 455), (476, 546), (28, 273), (56, 273), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (700, 364), (280, 455), (476, 273), (672, 91), (504, 273), (168, 637), (672, 182), (588, 273), (672, 273), (168, 91), (168, 182)$	21
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (280, 0), (280, 455), (588, 182), (252, 546), (672, 182), (588, 273), (56, 91), (700, 182), (56, 182), (700, 273), (168, 91), (448, 546), (560, 455), (588, 455), (56, 273)$	21
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (168, 0), (560, 364), (588, 364), (280, 455), (560, 182), (588, 182), (504, 273), (28, 91), (588, 273), (56, 91), (280, 637), (476, 455), (28, 182), (56, 182), (168, 91), (560, 455), (588, 455), (56, 273), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (476, 364), (168, 0), (560, 364), (672, 364), (476, 273), (672, 91), (504, 273), (168, 637), (280, 546), (28, 91), (56, 91), (280, 637), (476, 455), (168, 91), (448, 546), (560, 455), (476, 546), (504, 546), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (168, 0), (560, 364), (252, 0), (672, 364), (672, 91), (252, 546), (280, 546), (28, 91), (56, 91), (700, 182), (252, 637), (280, 637), (56, 182), (700, 273), (168, 91), (448, 546), (560, 455), (56, 273), (252, 91), (448, 637)$	14

Table 11: Two-intersection sets of type $(182, 6, 56, 65)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (504, 364), (168, 0), (560, 364), (560, 182), (168, 637), (560, 273), (28, 91), (56, 91), (252, 637), (448, 455), (280, 637), (28, 182), (504, 455), (168, 91), (448, 546), (588, 455), (504, 546), (56, 273), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (168, 0), (560, 364), (280, 0), (700, 364), (280, 455), (560, 182), (700, 91), (168, 637), (280, 546), (28, 91), (56, 91), (700, 182), (252, 637), (280, 637), (476, 455), (28, 182), (672, 273), (700, 273), (560, 455), (56, 273), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (504, 364), (168, 0), (560, 364), (672, 364), (700, 364), (560, 182), (672, 91), (504, 273), (560, 273), (28, 91), (672, 182), (700, 182), (448, 455), (28, 182), (504, 455), (168, 91), (28, 273), (588, 455), (504, 546), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (476, 364), (168, 0), (560, 364), (588, 364), (672, 364), (672, 91), (588, 182), (504, 273), (700, 91), (168, 637), (280, 546), (672, 182), (448, 455), (672, 273), (504, 455), (700, 273), (168, 91), (476, 546), (504, 546), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (168, 0), (560, 364), (252, 0), (280, 0), (672, 364), (700, 364), (280, 455), (560, 182), (700, 91), (252, 546), (280, 546), (672, 182), (588, 273), (56, 91), (700, 182), (252, 637), (28, 182), (672, 273), (700, 273), (588, 455), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (560, 364), (280, 0), (280, 455), (560, 182), (476, 273), (280, 546), (560, 273), (700, 182), (252, 637), (280, 637), (476, 455), (28, 182), (448, 546), (560, 455), (504, 546), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (168, 0), (560, 364), (588, 364), (280, 0), (672, 364), (700, 364), (280, 455), (672, 91), (588, 182), (504, 273), (700, 91), (28, 91), (672, 182), (588, 273), (672, 273), (56, 182), (168, 91), (560, 455), (588, 455), (504, 546), (56, 273)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (476, 364), (168, 0), (560, 364), (252, 0), (672, 364), (476, 273), (700, 91), (168, 637), (560, 273), (28, 91), (56, 91), (700, 182), (252, 637), (28, 182), (672, 273), (56, 182), (168, 91), (28, 273), (168, 182), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (504, 364), (168, 0), (560, 364), (476, 273), (588, 182), (504, 273), (252, 546), (168, 637), (280, 546), (560, 273), (56, 91), (252, 637), (476, 455), (504, 455), (168, 91), (560, 455), (504, 546), (56, 273), (252, 91), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (476, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (280, 455), (476, 273), (672, 91), (588, 182), (700, 91), (588, 273), (280, 637), (476, 455), (672, 273), (700, 273), (448, 546), (476, 546), (588, 455), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (168, 0), (560, 364), (280, 0), (280, 455), (476, 273), (672, 91), (700, 91), (252, 546), (168, 637), (28, 91), (672, 182), (700, 182), (280, 637), (476, 455), (28, 182), (672, 273), (700, 273), (168, 91), (28, 273), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (476, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (560, 182), (588, 182), (504, 273), (700, 91), (560, 273), (672, 182), (588, 273), (252, 637), (28, 182), (504, 455), (700, 273), (560, 455), (588, 455), (252, 91)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (476, 364), (168, 0), (560, 364), (672, 364), (700, 364), (280, 455), (476, 273), (672, 91), (168, 637), (28, 91), (700, 182), (252, 637), (280, 637), (476, 455), (28, 182), (672, 273), (168, 91), (448, 546), (28, 273), (252, 91)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (280, 0), (700, 364), (280, 455), (672, 91), (700, 91), (28, 91), (280, 637), (28, 182), (672, 273), (56, 182), (700, 273), (448, 546), (476, 546), (28, 273), (504, 546)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (448, 364), (168, 0), (560, 364), (252, 0), (700, 364), (672, 91), (700, 91), (252, 546), (560, 273), (28, 91), (588, 273), (700, 182), (448, 455), (672, 273), (56, 182), (700, 273), (560, 455), (28, 273), (588, 455), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (56, 0), (168, 0), (560, 364), (672, 364), (700, 91), (252, 546), (168, 637), (560, 273), (28, 91), (700, 182), (448, 455), (476, 455), (28, 182), (672, 273), (168, 91), (448, 546), (476, 546), (28, 273), (56, 273), (252, 91), (168, 182)$	14*
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (476, 364), (168, 0), (560, 364), (700, 364), (280, 455), (476, 273), (588, 182), (168, 637), (280, 546), (560, 273), (28, 91), (56, 91), (476, 455), (28, 182), (56, 182), (700, 273), (476, 546), (28, 273), (588, 455), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (448, 364), (168, 0), (560, 364), (252, 0), (560, 182), (672, 91), (504, 273), (700, 91), (168, 637), (28, 91), (672, 182), (588, 273), (672, 273), (504, 455), (700, 273), (168, 91), (448, 546), (28, 273), (588, 455), (504, 546), (168, 182)$	14

Table 11: Two-intersection sets of type $(182, 6, 56, 65)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (476, 364), (168, 0), (588, 364), (280, 0), (672, 364), (476, 273), (700, 91), (168, 637), (28, 91), (588, 273), (56, 91), (700, 182), (280, 637), (28, 182), (672, 273), (504, 455), (56, 182), (168, 91), (28, 273), (504, 546), (168, 182)$	14*
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (476, 364), (168, 0), (588, 364), (672, 364), (476, 273), (672, 91), (504, 273), (252, 546), (28, 91), (252, 637), (476, 455), (28, 182), (56, 182), (168, 91), (448, 546), (476, 546), (28, 273), (588, 455), (504, 546), (56, 273), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (504, 364), (168, 0), (588, 364), (476, 273), (168, 637), (280, 546), (28, 91), (672, 182), (700, 182), (280, 637), (28, 182), (672, 273), (504, 455), (56, 182), (700, 273), (168, 91), (476, 546), (28, 273), (588, 455), (56, 273), (168, 182)$	14
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (476, 364), (504, 364), (560, 364), (588, 364), (280, 0), (672, 364), (560, 182), (476, 273), (588, 182), (168, 637), (280, 546), (560, 273), (672, 182), (252, 637), (280, 637), (28, 182), (504, 455), (168, 91), (476, 546), (504, 546), (448, 637)$	14
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (560, 364), (588, 364), (672, 364), (476, 273), (588, 182), (252, 546), (560, 273), (672, 182), (28, 182), (672, 273), (56, 182), (168, 91), (28, 273), (588, 455), (56, 273), (252, 91), (168, 182), (448, 637)$	21
$(0, 0), (0, 91), (0, 182), (0, 273), (28, 0), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (672, 364), (280, 455), (476, 273), (672, 91), (504, 273), (168, 637), (280, 546), (672, 182), (588, 273), (700, 182), (672, 273), (56, 182), (168, 91), (448, 546), (168, 182)$	21
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (560, 364), (588, 364), (672, 364), (672, 91), (588, 182), (252, 546), (28, 91), (672, 182), (56, 91), (448, 455), (476, 455), (28, 182), (56, 182), (168, 91), (560, 455), (588, 455), (252, 91), (168, 182)$	21
$(0, 0), (0, 91), (0, 182), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (560, 364), (588, 364), (672, 364), (588, 182), (252, 546), (672, 182), (252, 637), (448, 455), (476, 455), (28, 182), (672, 273), (56, 182), (168, 91), (560, 455), (28, 273), (588, 455), (56, 273), (168, 182)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (700, 364), (476, 273), (672, 91), (700, 91), (168, 637), (560, 273), (280, 637), (504, 455), (28, 273), (588, 455), (56, 273), (252, 91), (448, 637)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (588, 364), (252, 0), (700, 364), (280, 455), (560, 182), (672, 91), (700, 91), (168, 637), (280, 546), (672, 182), (588, 273), (56, 91), (448, 455), (476, 455), (504, 455), (560, 455), (28, 273), (504, 546), (252, 91)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (700, 91), (252, 546), (168, 637), (560, 273), (672, 182), (56, 91), (700, 182), (700, 182), (280, 637), (28, 182), (448, 546), (476, 546), (588, 455), (504, 546)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (560, 364), (280, 0), (672, 364), (700, 364), (588, 182), (504, 273), (252, 546), (280, 546), (672, 182), (588, 273), (56, 91), (700, 182), (252, 637), (28, 182), (448, 546), (560, 455), (476, 546), (504, 546), (168, 182)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (560, 364), (588, 364), (252, 0), (672, 364), (280, 455), (560, 182), (504, 273), (700, 91), (672, 182), (280, 637), (28, 182), (504, 455), (56, 182), (700, 273), (168, 91), (448, 546), (476, 546), (588, 455), (252, 91)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (252, 0), (280, 0), (700, 364), (560, 182), (588, 182), (280, 546), (560, 273), (672, 182), (588, 273), (700, 182), (252, 637), (672, 273), (56, 182), (476, 546), (28, 273), (504, 546), (168, 182), (448, 637)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (672, 364), (700, 364), (560, 182), (672, 91), (252, 546), (280, 546), (700, 182), (280, 637), (476, 455), (28, 182), (504, 455), (56, 182), (168, 91), (448, 546), (588, 455)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (280, 0), (476, 273), (504, 273), (252, 546), (168, 637), (560, 273), (672, 182), (56, 91), (700, 182), (252, 637), (280, 637), (672, 273), (700, 273), (448, 546), (28, 273), (588, 455), (448, 637)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (504, 364), (168, 0), (252, 0), (280, 455), (560, 182), (588, 182), (504, 273), (700, 91), (168, 637), (280, 546), (560, 273), (28, 91), (672, 182), (588, 273), (700, 182), (448, 455), (476, 455), (672, 273), (476, 546), (56, 273), (252, 91)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (700, 364), (476, 273), (672, 91), (168, 637), (560, 273), (28, 91), (588, 273), (252, 637), (280, 637), (672, 273), (700, 273), (448, 546), (28, 273), (588, 455), (448, 637)$	21

Table 11: Two-intersection sets of type $(182, 6, 56, 65)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (504, 364), (168, 0), (252, 0), (280, 455), (560, 182), (476, 273), (588, 182), (700, 91), (252, 546), (560, 273), (672, 182), (588, 273), (280, 637), (476, 455), (28, 182), (672, 273), (56, 182), (700, 273), (448, 546), (504, 546), (168, 182)$	20160
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (700, 364), (560, 273), (588, 273), (448, 455), (280, 637), (476, 455), (672, 273), (504, 455), (700, 273), (168, 91), (28, 273), (56, 273), (252, 91)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (476, 364), (168, 0), (560, 364), (280, 0), (672, 364), (700, 364), (280, 455), (476, 273), (672, 91), (588, 182), (504, 273), (252, 546), (168, 637), (28, 91), (588, 273), (56, 91), (252, 637), (700, 273), (448, 546), (560, 455), (504, 546), (448, 637)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (280, 0), (476, 273), (672, 91), (504, 273), (700, 91), (252, 546), (168, 637), (560, 273), (672, 182), (56, 91), (700, 182), (448, 455), (280, 637), (448, 455), (280, 637), (448, 546), (28, 273), (588, 455), (252, 91)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (504, 364), (168, 0), (252, 0), (560, 182), (476, 273), (588, 182), (280, 546), (560, 273), (28, 91), (672, 182), (588, 273), (56, 91), (700, 182), (448, 455), (280, 637), (672, 273), (504, 455), (700, 273), (168, 91), (476, 546), (252, 91)$	42
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (588, 364), (252, 0), (700, 364), (280, 455), (560, 182), (476, 273), (672, 91), (504, 273), (700, 91), (168, 637), (280, 546), (672, 182), (588, 273), (252, 637), (448, 455), (560, 455), (28, 273), (504, 546), (56, 273)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (700, 364), (280, 455), (504, 273), (700, 91), (28, 91), (56, 91), (476, 455), (672, 273), (168, 91), (560, 455), (588, 455), (252, 91), (448, 637)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (280, 0), (700, 91), (252, 546), (168, 637), (560, 273), (28, 91), (672, 182), (56, 91), (700, 182), (252, 637), (280, 637), (476, 455), (672, 273), (504, 455), (448, 546), (588, 455), (448, 637)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (476, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (700, 364), (280, 455), (476, 273), (504, 273), (700, 91), (560, 273), (28, 91), (672, 182), (588, 273), (672, 273), (168, 91), (448, 546), (504, 546), (56, 273), (252, 91), (448, 637)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (168, 0), (280, 0), (280, 455), (560, 182), (476, 273), (588, 182), (700, 91), (252, 546), (28, 91), (672, 182), (56, 91), (700, 182), (252, 637), (448, 455), (672, 273), (504, 455), (168, 91), (560, 455), (476, 546), (588, 455), (504, 546)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (252, 0), (280, 0), (700, 364), (560, 182), (588, 182), (280, 546), (560, 273), (28, 91), (672, 182), (588, 273), (700, 182), (448, 455), (672, 273), (56, 182), (476, 546), (504, 546), (252, 91), (168, 182)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (700, 364), (280, 455), (476, 273), (504, 273), (168, 637), (560, 273), (28, 91), (252, 637), (672, 273), (700, 273), (588, 455), (56, 273), (448, 637)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (280, 0), (280, 455), (476, 273), (672, 91), (504, 273), (252, 546), (672, 182), (56, 91), (700, 182), (448, 455), (700, 273), (168, 91), (448, 546), (560, 455), (28, 273), (588, 455), (252, 91)$	21*
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (252, 0), (280, 0), (700, 364), (672, 91), (252, 546), (280, 546), (560, 273), (588, 273), (56, 91), (700, 182), (28, 182), (672, 273), (504, 455), (168, 91), (448, 546), (560, 455), (476, 546), (588, 455)$	21
$(0, 0), (0, 91), (28, 0), (56, 0), (476, 364), (504, 364), (168, 0), (560, 364), (252, 0), (672, 364), (700, 364), (672, 91), (588, 182), (700, 91), (280, 546), (28, 91), (588, 273), (56, 91), (280, 637), (476, 455), (504, 455), (168, 91), (448, 546), (560, 455), (252, 91), (448, 637)$	42
$(0, 0), (0, 91), (28, 0), (56, 0), (476, 364), (168, 0), (560, 364), (280, 0), (672, 364), (700, 364), (280, 455), (672, 91), (588, 182), (504, 273), (700, 91), (252, 546), (28, 91), (588, 273), (56, 91), (252, 637), (476, 455), (168, 91), (448, 546), (560, 455), (504, 546), (448, 637)$	42
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (672, 364), (700, 364), (560, 182), (672, 91), (588, 182), (504, 273), (700, 91), (252, 546), (280, 546), (560, 273), (28, 91), (588, 273), (56, 91), (252, 637), (448, 455), (280, 637), (476, 455), (168, 91), (504, 546)$	14
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (168, 0), (280, 0), (280, 455), (560, 182), (476, 273), (588, 182), (504, 273), (252, 546), (560, 273), (28, 91), (672, 182), (588, 273), (56, 91), (700, 182), (252, 637), (448, 455), (672, 273), (700, 273), (168, 91), (476, 546), (504, 546)$	42

Table 11: Two-intersection sets of type $(182, 6, 56, 65)$

Segre variety representation	Stabiliser size
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (168, 0), (252, 0), (280, 0), (672, 364), (280, 455), (560, 182), (672, 91), (588, 182), (504, 273), (560, 273), (28, 91), (588, 273), (56, 91), (700, 182), (448, 455), (476, 455), (700, 273), (168, 91), (504, 546), (252, 91)$	42
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (700, 364), (280, 455), (672, 91), (700, 91), (28, 91), (56, 91), (448, 455), (476, 455), (504, 455), (168, 91), (560, 455), (588, 455), (252, 91)$	546**
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (168, 0), (560, 364), (588, 364), (476, 273), (504, 273), (252, 546), (280, 546), (28, 91), (672, 182), (56, 91), (700, 182), (252, 637), (448, 455), (280, 637), (672, 273), (700, 273), (168, 91), (560, 455), (476, 546), (588, 455), (504, 546)$	14
$(0, 0), (0, 91), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (560, 364), (588, 364), (672, 364), (700, 364), (280, 455), (588, 182), (168, 637), (672, 182), (700, 182), (252, 637), (280, 637), (28, 182), (504, 455), (56, 182), (168, 91), (448, 546), (560, 455), (476, 546), (252, 91)$	168
$(0, 0), (0, 182), (28, 0), (56, 0), (448, 364), (476, 364), (504, 364), (168, 0), (560, 364), (588, 364), (252, 0), (280, 0), (672, 364), (700, 364), (560, 182), (588, 182), (252, 546), (280, 546), (672, 182), (700, 182), (28, 182), (56, 182), (448, 546), (476, 546), (504, 546), (168, 182)$	1092**
$(0, 0), (0, 182), (28, 0), (448, 364), (504, 364), (168, 0), (588, 364), (700, 364), (280, 455), (476, 273), (672, 91), (588, 182), (560, 273), (56, 91), (700, 182), (252, 637), (280, 637), (476, 455), (28, 182), (672, 273), (448, 546), (560, 455), (504, 546), (56, 273), (252, 91), (168, 182)$	84

The above examples give rise to strongly regular graphs with parameters $(729, 364, 181, 182)$. The example with stabiliser of order 20160 arises by taking the derived subgroup of the stabiliser of the Hill-cap. This group has orbit lengths 56, 56, 126, 126 on the points of $\text{PG}(5, 3)$. Taking a union of an orbit of size 56 with one of size 126 yields a two-intersection set with parameters $(182, 6, 56, 65)$. The five examples in the table above that have an asterisk in the last column arise from partial spreads of $\text{PG}(5, 3)$; the ‘SU2’ construction in [2, §7]. Two of these examples arise from the ‘CY2’ construction in [2, §9], denoted by two asterisks.

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